

# Event rates vs. cross sections at neutrino telescopes

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One of the major goals of neutrino astronomy is to explore the otherwise unknown fluxes and interactions of ultrahigh energy neutrinos. The existing neutrino telescopes look at three types of events: particle showers, muons, and taus. In this paper we discuss the dependence of the event rates on the neutrino nucleon cross-sections as we scale the cross sections, with energy, in different scenarios beyond the standard model. Our focus will be on the IceCube detector.

*Introduction:* Neutrino-nucleon cross sections at ultrahigh energies are unknown at present. These cross sections might deviate from their standard model (SM) values due to the turn on of any new-physics processes above a TeV center of mass energies. A number of neutrino telescopes are already taking data and others are under developmental stage. One of the major goals of these telescopes is to determine neutrino-nucleon cross sections above a PeV. As the neutrino fluxes are not known either, the cross section information is lost if one merely looks at the absolute event rates integrated over neutrino energy. To determine the cross sections one needs to calculate the ratios of the upward and downward event rates. The knowledge of the scaling of event rates with cross sections also helps differentiate among the cross section models. In this work we study the scaling of neutrino event rates with the cross sections. For details see Ref. [1]. A number of authors have done similar studies [2, 3, 4, 5, 6].

*A simple analytic model:* Here we give a very simple model for the dependence of neutrino event rates on the neutrino-nucleon cross sections. We will discuss the case for two generic detectors: an underground volume detector, like IceCube [7] and a planar detector placed horizontally on the surface of the Earth. The cross section dependence of the *downward* event rates is trivial. The *downward* shower rates scale as  $\frac{d\Gamma}{dE_\nu} \sim \sigma_t$ , where  $\sigma_t$  is the total cross section due to all the processes that produce showers. Similarly, the lepton event rates are given by  $\frac{d\Gamma}{dE_\nu} \sim \sigma_{cc}$ , where  $\sigma_{cc}$  is the charged current (CC) interaction cross-section.

The dependence of the *upward* rates on the cross sections is derived as follows: **i)** the rates depend on the upward flux at the detector:  $\frac{d\Gamma}{dE_\nu} \sim F_d$ , where  $F_d$  depends on the attenuation cross section. If one ignores both the down-scattered neutrinos and the coupling among different neutrino flavors due to tau decays, the flux at the detector at nadir angle  $\theta$  is  $F_d = F_0 e^{-l/\lambda_a}$ . Here  $F_0$  is the flux falling on surface of the Earth;  $l$  is the length neutrino travels through the Earth to reach the detector, and  $\lambda_a = 1/N_A \rho \sigma_a$  ( $N_A$  is Avogadro's number and  $\rho$  is the density of the Earth which is assumed to be constant in our simple model).  $\sigma_a$  is the attenuation cross section defined by  $\sigma_a = \sum_i y_i \sigma_i$ . The sum is over different types of interactions that a neutrino undergoes during its propagation through the Earth; here  $y$  is the average inelasticity of an interaction defined as:  $y = 1$  for a  $\nu_e$  or  $\nu_\mu$  charged current interaction; for  $\nu_\tau$  charged current interactions, although a neutrino is completely absorbed, it is regenerated by tau decays; hence the effective  $y$  for  $\nu_\tau$ 's is smaller than 1. For neutral current interactions  $y$  is just the conventional average inelasticity of the interaction ( $y \simeq 0.2 - 0.25$  [8]). Given these considerations, for standard model,  $\sigma_a(SM) \simeq 0.7\sigma_t(SM)$  around 10 PeV [1]. **ii)** The rates depend on the interaction cross section:  $\frac{d\Gamma}{dE_\nu} \sim \sigma_i$ ; for showers,  $\sigma_i$  is the total cross section due to the processes that produce showers and for leptons it is the charged current interaction cross section. **iii)** The rates depend on the area projected perpendicular to the direction of incoming neutrino:  $\frac{d\Gamma}{dE_\nu} \sim A_p$ . For a volume detector,  $A_p$  is almost a constant; for a surface detector,  $A_p$  is a function of the nadir angle; for example, for a planar detector sitting horizontally on the surface of Earth, the projected area is  $A_p = A \cos\theta$ , where  $A$  is the surface area of the detector. This difference in  $A_p$  for a volume detector and a planar detector causes the rates, integrated over angle, to be different for the detectors as we discuss below.

The above discussion implies  $\frac{d\Gamma}{dE_\nu} \sim \frac{1}{\lambda_i} \int d\Omega A_p F_d \sim \frac{F_0}{\lambda_i} \int_0^{\pi/2} d\theta \sin\theta A_p e^{-l/\lambda_a}$  which for a volume detector, placed on the surface (depth  $d = 0$ ) in our simple model, reduces to  $\frac{d\Gamma}{dE_\nu} \sim \frac{\lambda_a}{\lambda_i} (1 - e^{-2R/\lambda_a})$ , and for a planar detector placed horizontally on the surface of Earth it reduces to  $\frac{d\Gamma}{dE_\nu} \sim \frac{\lambda_a^2}{\lambda_i} (1 - e^{-2R/\lambda_a} - \frac{2R}{\lambda_a} e^{-2R/\lambda_a})$ . Now we can write the scaling expressions for the event rates in two different limits: For  $\lambda_i \ll 2R$ , we get:  $\frac{d\Gamma}{dE_\nu} \sim \frac{\lambda_a}{\lambda_i} = \sigma_i/\sigma_a$  for a volume detector and  $\frac{d\Gamma}{dE_\nu} \sim \frac{\lambda_a^2}{\lambda_i} = \sigma_i/\sigma_a^2$  for the planar detector. For  $\lambda_i \gg 2R$ , the Earth becomes transparent and both for volume and planar detectors we get  $\frac{d\Gamma}{dE_\nu} \sim \sigma_i$  (same as downward event rates as expected).

*Detailed Simulation:* We solve coupled Boltzmann transport equations for three neutrino flavors and the taus [9].

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As an example, we use IceCube as our detector and use the WB model flux  $F_0 = 6 \times 10^{-8} (\text{GeV} \cdot \text{cm}^{-2} \cdot \text{s}^{-1}) / (E(\text{GeV}))^2$  [10]. We include tau decay and the down scattering terms in the Boltzmann transport equations. We count all showers produced within the detector volume and take the energy losses of muons and taus to be independent of their energies; we also ignore tagging of taus here; our arguments about the scaling of event rates are not sensitive to these details. We do the simulation assuming three cases for neutrino-nucleon cross section models as explained below. Figures 1 and 2 show the results obtained from detailed simulation. For details see Ref. [1].

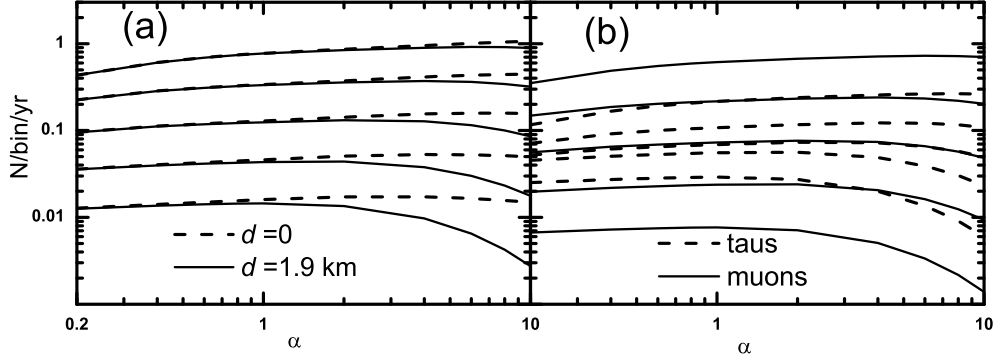


FIG. 1: Expected event rates in IceCube vs.  $\alpha$  (see text). Pairs of curves, from top to bottom, are for  $10^{6.5} \leq E_\nu \leq 10^9$  GeV in five bins of width  $\Delta \log_{10} E_\nu = 0.5$ . Fig 1(a) shower rates while the Fig. 1(b) shows lepton rates with  $d=1.9$  km.

Fig. 1 shows the results for the case where we assume the following cross-section models:  $\sigma_{cc} = \alpha \sigma_{cc}(SM)$  and  $\sigma_{nc} = \alpha \sigma_{NC}(SM)$  (these models are motivated by the fact that the QCD saturation effects might alter the scaling of neutrino-nucleon cross sections with energy at ultrahigh energies [11]). For this case,  $\sigma_a = (1 + \alpha) \sigma_a(SM)$  and  $\sigma_i$  [showers, leptons] =  $[(1 + \alpha) \sigma_t(SM), (1 + \alpha) \sigma_{cc}(SM)]$ . Therefore, our simple analytic model implies that the rates  $\frac{d\Gamma}{dE_\nu} \sim \sigma_i / \sigma_a$  are independent of  $\alpha$  for these cross section models. Let us compare this prediction with the detailed simulation shown in Fig. 1. Fig. 1(a) shows results for the shower rates for a detector with IceCube characteristics. To illustrate the importance of detector depth  $d$ , we show results for  $d = 1.9$  km (IceCube depth) and  $d = 0$  in Fig. 1(a). Fig. 1(b) shows lepton rates for  $d = 1.9$  km case. We see in Fig.1(a): **a)** For higher cross sections and energies, even a depth of 1.9 km makes a significant difference in the event rates as compared to the  $d = 0$  case. This is explained as follows: For  $d = 1.9$  km case, even for horizontal neutrinos the distance traveled through the Earth is  $l \simeq 220$  km; hence the rate  $\frac{d\Gamma}{dE_\nu} \sim \sigma_i e^{-220(km)/\lambda_a}$  is an upper limit on how fast the event rates can grow with the cross section for  $d = 1.9$  km case. At higher energies and cross sections,  $\lambda_a < 220(km)$  (e.g.  $\sigma(new) = 10\sigma(SM)$ ,  $E_\nu = 10^9$  GeV  $\Rightarrow \lambda_a \simeq 50$  km), hence the above upper limit on the rate is a decreasing function of  $\alpha$  for our cross section models. On the other hand, for  $d = 0$  case,  $\frac{d\Gamma}{dE_\nu} \sim \sigma_i / \sigma_a$  from our analytic model. These trends are obvious in Fig. 1(a). **b)** Another extreme limit is the one at low energies and cross sections (upper left corner in Fig. 1(a)) where both cases,  $d = 0$  and  $d = 1.9$  km, are indistinguishable. This is because, in this limit, the Earth is becoming transparent to the neutrinos. **c)** For the case between the ‘a’ and ‘b’ extreme cases, our analytic model gives  $\frac{d\Gamma}{dE_\nu} \sim \sigma_i / \sigma_a$ ; however, in Fig. 1, we see there is a slight increase in the rates with  $\alpha$  which is due to the details of the simulation that are absent from our analytic model (see the discussion of Fig. 2 below).

Fig. 2 compares the scaling of event rates with the cross sections, for the energy bin  $10^{6.5} \leq E_\nu \leq 10^7$  GeV, for different types of cross section models. The purpose of this figure is to illustrate the importance of inelasticity of the cross sections in the scaling arguments. All the results are for IceCube with  $d = 1.9$  km. There are three triplets of curves: upper, middle, and lower; these triplets correspond to shower, muon, and tau rates, respectively. Each triplet has three curves (dashed, solid, and dotted) corresponding to three classes of models (labeled in the legend in Fig. 2) for the cross sections. We use the following cross section models in Fig. 2: For the dotted curves, our cross section model is the same as in Fig. 1. For the solid curves (motivation is black hole formation in low scale gravity models [12]), in addition to the SM neutral current and charged current cross sections, we introduce a new cross section with inelasticity = 1 and we normalize it to the SM total cross section at  $\alpha = 1$ . This implies  $\sigma_a = \sigma_a(SM) + \alpha \sigma_t(SM)$  and  $\sigma_i$  [showers, leptons] =  $[(1 + \alpha) \sigma_t(SM), \sigma_{cc}(SM)] \Rightarrow \sigma_i / \sigma_a$  [showers, leptons]  $\cong [(1 + \alpha) / (0.67 + \alpha), 0.7 / (0.67 + \alpha)]$ . For the dashed curves (motivation is graviton exchange in low scale gravity models.), in addition to the SM neutral current and charged current cross sections, we introduce a cross section equivalent to the SM neutral current cross section and we normalize it to the SM total cross section at  $\alpha = 1$ . This implies  $\sigma_a = \sigma_a(SM) + \alpha 0.25 \sigma_t(SM)$  and  $\sigma_i$  [showers, leptons] =  $[(1 + \alpha) \sigma_t(SM), \sigma_{cc}(SM)] \Rightarrow \sigma_i / \sigma_a$  [showers, leptons]  $\cong [(1 + \alpha) / (0.67 + 0.25\alpha), 0.7 / (0.67 + 0.25\alpha)]$ .

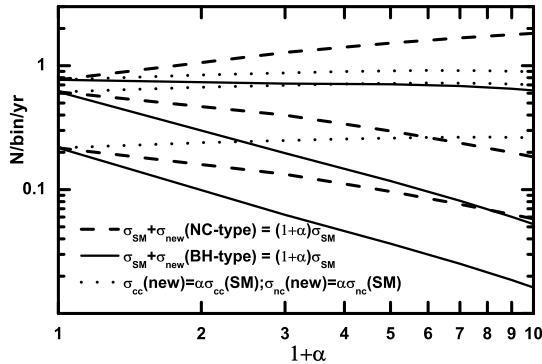


FIG. 2: Comparison of rates, for the bin  $10^{6.5} \leq E_\nu \leq 10^7$  GeV, for different cross section models. There are three triplets of curves: Top: showers. Middle: muons. Bottom: taus.

Fig. 2 is in good agreement with our simple analytic model; the event rates scale like  $\sigma_i/\sigma_a$ . The slight deviations from the expected scaling law ( $\frac{d\Gamma}{dE_\nu} \sim \sigma_i/\sigma_a$ ) are due to a couple of details that we have ignored in our simple model but have included in the simulation (unlike the simulation, our simple model assumes constant values for Earth density and cross section inelasticity, and it also ignores the down scattering of neutrinos).

In the above study, the normalization of the event rates was not our concern. To get an estimate of the event rates in the SM for IceCube, we have done a more realistic simulation which includes energy dependent energy losses for muons and taus; for tau energy losses we use approximation III from Ref. [13] and we use energy dependent effective area and effective volume for IceCube as given in Ref. [6]. For WB model, we get the upward rates per year in the energy bin  $10^{6.5} \leq E_\nu \leq 10^7$  GeV: *showers* = 0.7, *muons* = 0.8, and *taus* = 0.2 (without tag); tau rate with tagging (as defined in Ref. [9]) is 0.1. The rates integrated over energy above a PeV are: *showers* = 2.4, *muons* = 2.2, *taus* = 0.9 (without tag), and *taus* with tag = 0.3.

*Summary:* We have shown that, in general, the cross section dependence of the event rates depends on the event type, neutrino energy, and the cross-section model. **a)** Downward shower rate is  $\frac{d\Gamma}{dE_\nu} \sim \sigma_t$ ; downward lepton rate is  $\frac{d\Gamma}{dE_\nu} \sim \sigma_{cc}$ . **b)** At neutrino energies around 1-10 PeV and for a volume detector, within a decade of the change in cross sections around the SM value, upward shower rate is  $\frac{d\Gamma}{dE_\nu} \sim \sigma_t/\sigma_a$  and the upward lepton rate is  $\sim \sigma_{cc}/\sigma_a$ . For a surface detector, the upward lepton rate is  $\frac{d\Gamma}{dE_\nu} \sim \sigma_{cc}/\sigma_a^2$ . **c)** For energies around a PeV and cross sections around 0.1 of the SM value, the Earth becomes transparent and the upward rates scale with the cross-section like the downward rates. **d)** As one goes to higher energies and/or cross sections, the detector depth becomes important; even a depth of  $d = 2$  km can give a significantly different scaling of the rates with the cross sections, as compared to the  $d = 0$  case (Fig. 1(a)). **e)** Scaling of the rates with the cross sections is significantly different for different models of cross sections (Figs. 2).

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